RELATIONS AND FUNCTIONS

BASIC CONCEPTS

1. **Relation:** If A and B are two non-empty sets, then any subset R of $A \times B$ is called relation from set A to set B.

   i.e., $$R : A \to B \Leftrightarrow R \subseteq A \times B$$

   If $(x, y) \in R$, then we write $x\, R\, y$ (read as x is R related to y) and if $(x, y) \notin R$, then we write $x \not R\, y$ (read as x is not R related to y).

2. **Domain and Range of a Relation:** If R is any relation from set A to set B then,

   (*a*) **Domain of** R **is** the set of all first coordinates of elements of R and it is denoted by Dom (R).

   (*b*) **Range of** R **is** the set of all second coordinates of R and it is denoted by Range (R).

   A relation R on set A means, the relation from A to A *i.e.*, $R \subseteq A \times A$.

3. **Some Standard Types of Relations:**

   Let A be a non-empty set. Then, a relation R on set A is said to be

   (*a*) **Reflexive:** If $(x, x) \in R$ for each element $x \in A$, *i.e.*, if xRx for each element $x \in A$.

   (*b*) **Symmetric:** If $(x, y) \in R \Rightarrow (y, x) \in R$ for all $x, y \in A$, *i.e.*, if $xRy \Rightarrow yRx$ for all $x, y \in A$.

   (*c*) **Transitive:** If $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ for all $x, y, z \in A$, *i.e.*, if xRy and $yRz \Rightarrow xRz$.

4. **Equivalence Relation:** Any relation R on a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

5. **Antisymmetric Relation:** A relation R in a set A is antisymmetric

   if $(a, b) \in R, (b, a) \in R \Rightarrow a = b \ \forall\ a, b \in R$, or aRb and $bRa \Rightarrow a = b, \forall\ a, b \in R$.

   For example, the relation "greater than or equal to, "≥" is antisymmetric relation as

   $$a \geq b, b \geq a \Rightarrow a = b \ \forall\ a, b$$

   [Note: "Antisymmetric" is completely different from not symmetric.]

6. **Equivalence Class:** Let R be an equivalence relation on a non-empty set A. For all $a \in A$, the equivalence class of 'a' is defined as the set of all such elements of A which are related to 'a' under R. It is denoted by $[a]$.

   i.e., $$[a] = \text{equivalence class of } 'a' = \{x \in A : (x, a) \in R\}$$

7. **Function:** Let X and Y be two non-empty sets. Then, a rule f which associates to each element $x \in X$, a unique element, denoted by $f(x)$ of Y, is called a function from X to Y and written as $f : X \to Y$ where, $f(x)$ is called image of x and x is called the **pre-image** of $f(x)$ and the set Y is called the **co-domain** of f and $f(X) = \{f(x): x \in X\}$ is called the range of f.

8. **Types of Function:**

(*i*) **One-one function (injective function):** A function $f : X \to Y$ is defined to be one-one if the image of distinct element of X under rule f are distinct, *i.e.*, for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies that $x_1 = x_2$.

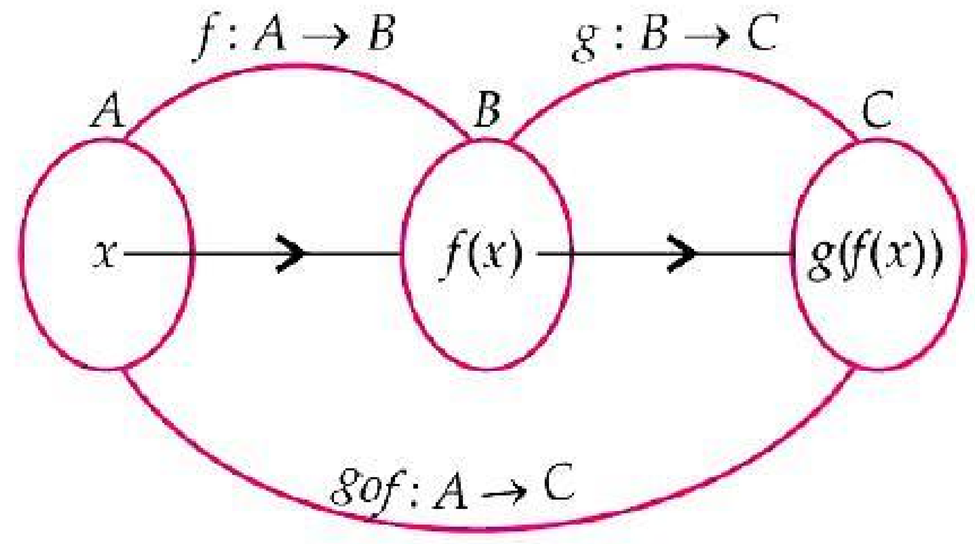
(*ii*) **Onto function (Surjective function):** A function $f : X \to Y$ is said to be onto function if each element of Y is the image of some element of x *i.e.*, for every $y \in Y$, there exists some $x \in X$, such that $y = f(x)$. Thus f is onto if range of f = co-domain of f.

(*iii*) **One-one onto function (Bijective function):** A function $f : X \to Y$ is said to be one-one onto, if f is both one-one and onto.

(*iv*) **Many-one function:** A function $f : X \to Y$ is said to be a many-one function if two or more elements of set X have the same image in Y. *i.e.*,

$f : X \to Y$ is a many-one function if there exist $a, b \in X$ such that $a \neq b$ but $f(a) = f(b)$.

9. **Composition of Functions:** Let $f : A \to B$ and $g : B \to C$ be two functions. Then, the composition of f and g, denoted by gof, is defined as the function.



$gof : A \to C$ given by

$gof(x) = g(f(x)), \forall\ x \in A$

Clearly, dom(gof) = dom(f)

Also, gof is defined only when range(f) $\subseteq$ dom(g)

10. **Identity Function:** Let R be the set of real numbers. A function $I : R \to R$ such that

$I(x) = x\ \forall\ x \in R$ is called identity function.

Obviously, identity function associates each real number to itself.

11. **Invertible Function:** For $f : A \to B$, if there exists a function $g : B \to A$ such that $gof = I_A$ and $fog = I_B$, where I_A and I_B are identity functions, then f is called an invertible function, and g is called the inverse of f and it is written as $f^{-1} = g$.

12. **Number of Functions:** If X and Y are two finite sets having m and n elements respectively then the number of functions from X to Y is n^m.

13. **Vertical Line Test:** It is used to check whether a relation is a function or not. Under this test, graph of given relation is drawn assuming elements of domain along x-axis. If a vertical line drawn anywhere in the graph, intersects the graph at only one point then the relation is a function, otherwise it is not a function.

14. **Horizontal Line Test:** It is used to check whether a function is one-one or not. Under this test graph of given function is drawn assuming elements of domain along x-axis. If a horizontal line (parallel to x-axis) drawn anywhere in graph, intersects the graph at only one point then the function is one-one, otherwise it is many-one.

MULTIPLE CHOICE QUESTIONS

Choose and write the correct option in the following questions.

1. **The relation R in the set $A = \{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$ is**
   (*a*) reflexive and symmetric but not transitive
   (*b*) reflexive and transitive but not symmetric
   (*c*) symmetric and transitive but not reflexive
   (*d*) an equivalence relation
2. **If $A = \{a, b, c, d\}$, then a relation $R = \{(a, b), (b, a), (a, a)\}$ on A is**
   (*a*) symmetric only (*b*) transitive only
   (*c*) reflexive and transitive (*d*) symmetric and transitive only
3. **For real numbers x and y, define xRy if and only if $x - y + \sqrt{2}$ is an irrational number. Then the relation R is** [*NCERT Exemplar*]
   (*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) none of these
4. **Consider the non-empty set consisting of children in a family and a relation R defined as aRb if a is brother of b. Then R is** [*NCERT Exemplar*]
   (*a*) symmetric but not transitive (*b*) transitive but not symmetric
   (*c*) neither symmetric nor transitive (*d*) both symmetric and transitive
5. **The maximum number of equivalence relation on the set $A = \{1, 2, 3\}$ are** [*NCERT Exemplar*]
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 5
6. **Let L denotes the set of all straight lines in a plane. Let a relation R be defined by lRm if and only if l is perpendicular to m $\forall$ $l, m \in L$. Then R is** [*NCERT Exemplar*]
   (*a*) reflexive (*b*) symmetric (*c*) transitive (*d*) none of these
7. **Let $A = \{1, 2, 3\}$. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is**
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4
8. **Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is/are**
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4
9. **Let A and B be finite sets containing m and n elements respectively. The number of relations that can be defined from A to B is**
   (*a*) 2^{mn} (*b*) 2^{m+n} (*c*) mn (*d*) 0
10. **Set A has 3 elements and the set B has 4 elements. Then the number of injective mapping that can be defined from A to B is** [*NCERT Exemplar*]
    (*a*) 144 (*b*) 12 (*c*) 24 (*d*) 64
11. **The function $f : R \to R$ defined by $f(x) = 2^x + 2^{|x|}$ is**
    (*a*) One-one and onto (*b*) Many-one and onto
    (*c*) One-one and into (*d*) Many-one and into
12. **If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mapping from A to B is**
    (*a*) 720 (*b*) 120 (*c*) 0 (*d*) none of these
13. **Which of the following functions from Z into Z is bijection?** [*NCERT Exemplar*]
    (*a*) $f(x) = x^3$ (*b*) $f(x) = x + 2$ (*c*) $f(x) = 2x + 1$ (*d*) $f(x) = x^2 + 1$
14. **Let $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is** [*NCERT Exemplar*]
    (*a*) R (*b*) $[1, \infty)$ (*c*) $[4, \infty)$ (*d*) $[5, \infty)$

15. **Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and –3, respectively, are** *[NCERT Exemplar]*

(*a*) $\phi, \{4, -4\}$ (*b*) $\{3, -3\}, \phi$ (*c*) $\{4, -4\}, \phi$ (*d*) $\{4, -4\}, \{2, -2\}$

16. **Let the function $f : R \to R$ be defined by $f(x) = 2x + \sin x$ for $x \in R$. Then f is**

(*a*) one-one but not onto (*b*) onto but not one-one
(*c*) neither one-one nor onto (*d*) one-one and onto

17. **Let R be the relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$ choose the correct answer.**

(*a*) $(2, 4) \in R$ (*b*) $(3, 8) \in R$ (*c*) $(6, 8) \in R$ (*d*) $(8, 7) \in R$

18. **Let $f : R \to R$ be defined as $f(x) = x^4$. Choose the correct answer**

(*a*) f is one-one onto (*b*) f is many one onto
(*c*) f is one-one but not onto (*d*) f is neither one-one nor onto.

19. **Let $f : R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.**

(*a*) f is one-one onto. (*b*) f is many one onto.
(*c*) f is one-one but not onto (*d*) f is neither one-one nor onto.

20. **Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by**
$f(x) = 2x^3 + 2x^2 + 300x + 5 \sin x$ then f is

(*a*) one-one onto (*b*) one-one into (*c*) many one onto (*d*) many one into

21. **Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-image of 5 and – 5, respectively are**

(*a*) $\phi, \{-2\}$ (*b*) $\{(3, -3\}, \phi$ (*c*) $\{-2, 2\}, \phi$ (*d*) $\{1, -1\}, \{2, -2\}$

22. **The domain of the function $f : R \to R$ defined by $f(x) = \sqrt{x^2 - 4}$ is**

(*a*) $[-2, 2]$ (*b*) $(-2, 2)$ (*c*) $(-\infty, -2] \cup [2, \infty)$ (*d*) $(-\infty, \infty)$

23. **Let $f : R \to R$ be defined by** $$f(x) = \begin{cases} 3x, & \text{if } x > 3 \\ x^2, & \text{if } 1 < x \le 3 \\ x, & \text{if } x \le 1 \end{cases}$$
Then $f(-2) + f(0) + f(2) + f(5)$ is equal to

(*a*) 0 (*b*) 17 (*c*) – 4 (*d*) none of these

24. **Let R is reflexive relation on a finite set A having n element, and let there be m ordered pairs in R. Then**

(*a*) $m \ge n$ (*b*) $m \le n$ (*c*) $m = n$ (*d*) none of these

25. **The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is**

(*a*) $(-3, -1) \cup (1, \infty)$ (*b*) $[-3, -1) \cup [1, \infty)$
(*c*) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ (*d*) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

26. **Let $f : R \to [0, \frac{\pi}{2})$ defined by $f(x) = \tan^{-1}(x^2 + x + a)$, then the set of values of a for which f is onto is**

(*a*) $[0, \infty)$ (*b*) $[\frac{1}{4}, \infty)$ (*c*) $[2, 1]$ (*d*) none of these

27. **If the function $f : R \to R$ and $g : R \to R$ are defined as**
$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$
and $$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$
then $(f - g)$ is

(*a*) one-one onto. (*b*) many-one onto
(*c*) one-one but not onto (*d*) neither one-one nor onto.

28. If a relation R on the set $\{1, 2, 3, 4\}$ is defined by $R = \{(1, 2), (3, 4)\}$. Then R is

(*a*) reflexive (*b*) transitive (*c*) symmetric (*d*) none of these

29. If the set A contains 4 elements and the set B contains 5 elements, then the number of one-one and onto mappings from A to B is

(*a*) 0 (*b*) 4^5 (*c*) 5^4 (*d*) none of these

30. Let $A = \{x, y, z)$ and $B = \{a, b\}$ then the number of onto function from A to B is

(*a*) 0 (*b*) 3 (*c*) 6 (*d*) 8

31. If A and B have 4 and 6 elements respectively then the number of one-one function from A to B is

(*a*) 4^6 (*b*) 6^4 (*c*) 360 (*d*) 240

32. If A and B have 4 elements each then the number of one-one onto (bijective) function from A to B is

(*a*) 0 (*b*) 24 (*c*) 4^2 (*d*) None of these

33. If R is an equivalence relation on A, then R^{-1} on A is

(*a*) Transitive only (*b*) Symmetric only (*c*) Reflexive only (*d*) Equivalence relation

34. The relation "greater than" denoted by > in the set of integers is

(*a*) Symmetric (*b*) Reflexive (*c*) Transitive (*d*) None of these

35. If R_1 and R_2 are symmetric relations in a set A, then $R_1 \cup R_2$ is

(*a*) Reflexive (*b*) Symmetric (*c*) Transitive (*d*) None of these

36. The function $f : R \rightarrow R$ defined by $f(x) = 4^x + 4^{|x|}$ is

(*a*) one-one and into (*b*) one-one and onto

(*c*) many one and into (*d*) many one and onto

37. Identity relation R on a set A is

(*a*) Reflexive only (*b*) Symmetric only (*c*) Transitive only (*d*) Equivalence

38. The relation "congruence modulo m" on the set $\mathbb{Z}$ of all integers is a relation of type

(*a*) Reflexive only (*b*) Symmetric only (*c*) Transitive only (*d*) Equivalence

39. Let $f : \mathbb{R} \longrightarrow \left[0, \frac{\pi}{2}\right)$ defined by $f(x) = \tan^{-1}(x^2 + x + 2a)$ then the set of values of 'a' for which f is onto, is

(*a*) $\left(-\frac{1}{4}, \infty\right)$ (*b*) $[-1, \infty)$ (*c*) $\left[-\frac{1}{8}, \infty\right)$ (*d*) $\left[\frac{1}{8}, \infty\right)$

40. If the function $f(x)$ satisfying $(f(x))^2 - 4f(x)f'(x) + (f'(x))^2 = 0$ then $f(x)$ equals

(*a*) $\lambda e^{(2+\sqrt{5})x}$ (*b*) $\lambda e^{(2-\sqrt{5})x}$ (*c*) $\lambda e^{(2\pm\sqrt{3})x}$ (*d*) $\lambda e^{(3-\sqrt{3})x}$

41. Let $f : (-1, 1) \longrightarrow B$ where $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is one-one and onto, then B equals

(*a*) $\left[0, \frac{\pi}{2}\right]$ (*b*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (*c*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (*d*) $\left(0, \frac{\pi}{2}\right)$

42. The function $y = \frac{x}{1+|x|}$, $x \in \mathbb{R}$, $y \in \mathbb{R}$ is

(*a*) One-one onto (*b*) Onto but not one-one

(*c*) One-one but not onto (*d*) None of these

43. A relation R in the set of non-zero complexs number is defined by $z_1 R z_2 \Leftrightarrow \frac{z_1 - z_2}{z_1 + z_2}$ is real, then R is

(*a*) Reflexive (*b*) Symmetric (*c*) Transitive (*d*) Equivalence

44. Number of onto (subjective) functions from A to B if $n(A) = 6$ and $n(B) = 3$ are

(*a*) $2^6 - 2$ (*b*) $3^6 - 3$ (*c*) 340 (*d*) None of these

45. **Let $A = \{7, 8, 9, 10\}$ and $R\ \{(8, 8), (9, 9), (10, 10), (7, 8)\}$ be a relation on A, then R is**

(*a*) Transitive (*b*) Reflexive (*c*) Symmetric (*d*) None of these

46. **Let f, g be a function from the set $\{1, 2, ..., 12\}$ to the set $\{1, 2, 3, ..., 11\}$ then which of the following is correct?**

(*a*) Number of onto functions from A to $B = \frac{\lfloor 12 \times 11}{2}$

(*b*) Total number of functions from A to $B = 11^{12}$

(*c*) The functions which are not onto $= 11^{12} - \frac{\lfloor 12 \times 11}{2}$

(*d*) All of these

47. **Let p and q are positive integers, f is a function defined for positive numbers and attains only positive values such that $f[x\ f(y)] = x^a y^b$ then**

(*a*) $b^2 = a$ (*b*) $a = b$ (*c*) $a^2 = b$ (*d*) None of these

48. **Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ then f is both one-one and onto when B is the interval**

(*a*) $\left[0, \frac{\pi}{2}\right)$ (*b*) $\left(0, \frac{\pi}{2}\right)$ (*c*) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (*d*) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

49. **Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ then**

(*a*) $f(x)$ is one-one but not onto (*b*) $f(x)$ is neither one-one nor onto

(*c*) $f(x)$ is many one but onto (*d*) $f(x)$ is one-one and onto

50. **If $A = \{7, 8, 9\}$, then the relation $R = \{(8, 9)\}$ in A is**

(*a*) Symmetric only (*b*) Non-symmetric (*c*) Reflexive only (*d*) Equivalence

51. **Let A be the finite set containing n distinct elements . The number of relations that can be defined on A is**

(*a*) 2^n (*b*) n^2 (*c*) 2^{n^2} (*d*) 2^{n-1}

52. **Let R_1 and R_2 be equivalence relations on a set A, then $R_1 \cup R_2$ may or may not be**

(*a*) Reflexive (*b*) Symmetric (*c*) Transitive (*d*) None of these

53. **Let R be the relation defined on the set N of natural numbers by the rule $x\ R\ y$ iff $x + 2y = 8$, then domain of R is**

(*a*) $\{2,4,8\}$ (*b*) $\{2,4,6\}$ (*c*) $\{2,4,6,8\}$ (*d*) $\{1,2,3,4\}$

54. **Let $A = \{a,b,c\}$ and $R = \{(a,a), (b,b), (c,c), (b,c), (a,b)\}$ be a relation on A, then R is**

(*a*) Symmetric (*b*) Transitive (*c*) Reflexive (*d*) Equivalence

55. **"Every relation is a function and every function is a relation" then which is correct for given statement ?**

(*a*) True (*b*) False (*c*) Can't say anything (*d*) None of these

56. **If a relation R on the set $\{1, 2, 3\}$ be defined by $R = \{(1, 2)\}$, then R is**

(*a*) Reflexive (*b*) Transitive (*c*) Symmetric (*d*) None of these

57. **Let us define a relation R in R as $a\ R\ b$ if $a \geq b$. Then, R is**

$R = \{(a, b) : a \geq b\}$

(*a*) An equivalence relation

(*b*) Reflexive, transitive but not symmetric

(*c*) Symmetric, transitive but not reflexive

(*d*) Neither transitive nor reflexive but symmetric

58. **If $A = \{1, 2, 3\}$ and consider the relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then, R is**

(*a*) Reflexive but not symmetric (*b*) Reflexive but not transitive

(*c*) Symmetric and transitive (*d*) Neither Symmetric nor transitive

59. **The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 7\}$ is given by**

(*a*) $\{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$

(*b*) $\{(2, 2), (3, 2), (4, 2), (2, 4)\}$

(*c*) $\{(3, 3), (4, 3), (5, 4), (3, 4)\}$

(*d*) $\{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2, 3)\}$

Answers

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*b*)	**5.** (*d*)	**6.** (*b*)
7. (*a*)	**8.** (*b*)	**9.** (*a*)	**10.** (*c*)	**11.** (*c*)	**12.** (*c*)
13. (*b*)	**14.** (*b*)	**15.** (*c*)	**16.** (*d*)	**17.** (*c*)	**18.** (*d*)
19. (*a*)	**20.** (*a*)	**21.** (*c*)	**22.** (*c*)	**23.** (*b*)	**24.** (*a*)
25. (*c*)	**26.** (*b*)	**27.** (*a*)	**28.** (*b*)	**29.** (*a*)	**30.** (*c*)
31. (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*c*)	**35.** (*b*)	**36.** (*a*)
37. (*d*)	**38.** (*d*)	**39.** (*d*)	**40.** (*c*)	**41.** (*c*)	**42.** (*c*)
43. (*d*)	**44.** (*d*)	**45.** (*a*)	**46.** (*d*)	**47.** (*c*)	**48.** (*c*)
49. (*b*)	**50.** (*b*)	**51.** (*c*)	**52.** (*c*)	**53.** (*b*)	**54.** (*c*)
55. (*b*)	**56.** (*b*)	**57.** (*b*)	**58.** (*a*)	**59.** (*d*)	

CASE-BASED QUESTIONS

Choose and write the correct option in the following questions.

1. **Read the following and answer any four questions from (*i*) to (*v*).**

A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever

ONE – NATION

ONE – ELECTION

FESTIVAL OF

DEMOCRACY

GENERAL ELECTION– 2019

Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

$R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in general election – 2019$\}$

[*CBSE Question Bank*]

Answer the questions given below.

(*i*) Two neighbours X and $Y \in I$. X exercised his voting right while Y did not cast her vote in general election – 2019. Which of the following is true?

(*a*) $(X, Y) \in R$ (*b*) $(Y, X) \in R$

(*c*) $(X, X) \notin R$ (*d*) $(X, Y) \notin R$

(*ii*) Mr.'X' and his wife 'W' both exercised their voting right in general election -2019, Which of the following is true?

(*a*) both (X,W) and $(W,X) \in R$ (*b*) $(X,W) \in R$ but $(W,X) \notin R$

(*c*) both (X,W) and $(W,X) \notin R$ (*d*) $(W,X) \in R$ but $(X,W) \notin R$

(*iii*) Three friends F_1, F_2 and F_3 exercised their voting right in general election- 2019, then which of the following is true?

(*a*) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

(*b*) $(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \notin R$

(*c*) $(F_1, F_2) \in R$, $(F_2, F_2) \in R$ but $(F_3, F_3) \notin R$

(*d*) $(F_1, F_2) \notin R$, $(F_2, F_3) \notin R$ and $(F_1, F_3) \notin R$

(*iv*) The above defined relation R is

(*a*) Symmetric and transitive but not reflexive

(*b*) Universal relation

(*c*) Equivalence relation

(*d*) Reflexive but not symmetric and transitive

(*v*) Mr. Shyam exercised his voting right in General Election – 2019, then Mr. Shyam is related to which of the following?

(*a*) All those eligible voters who cast their votes

(*b*) Family members of Mr.Shyam

(*c*) All citizens of India

(*d*) Eligible voters of India

Sol. We have a relation 'R' is defined on I as follows:

$R = \{V_1, V_2\} : V_1, V_2 \in I$ and both use their voting right in general election – 2019}

(*i*) Two neighbors X and $Y \in I$. Since X exercised his voting right while Y did not cast her vote in general election – 2019

Therefore, $(X, Y) \notin R$

$\therefore$ Option (*d*) is correct.

(*ii*) Since Mr. 'X' and his wife 'W' both exercised their voting right in general election – 2019.

$\therefore$ Both (X, W) and $(W, X) \in R$.

$\therefore$ Option (*a*) is correct.

(*iii*) Since three friends F_1, F_2 and F_3 exercised their voting right in general election – 2019, therefore

$(F_1, F_2) \in R$, $(F_2, F_3) \in R$ and $(F_1, F_3) \in R$

$\therefore$ Option (*a*) is correct.

(*iv*) This relation is an equivalence relation

$\therefore$ Option (*c*) is correct.

(*v*) Mr. Shyam exercised his voting right in General election – 2019, then Mr. Shyam is related to all those eligible votes who cast their votes.

$\therefore$ Option (*a*) is correct.

2. Read the following and answer any four questions from (*i*) to (*v*).

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set {1,2,3,4,5,6}. Let A be the set of players while B be the set of all possible outcomes.

$A = \{S, D\}$, $B = \{1,2,3,4,5,6\}$ [*CBSE Question Bank*]

Answer the questions given below.

(*i*) Let $R : B \to B$ be defined by $R = \{(x, y): y \text{ is divisible by } x\}$ is

(*a*) Reflexive and transitive but not symmetric

(*b*) Reflexive and symmetric and not transitive

(*c*) Not reflexive but symmetric and transitive

(*d*) Equivalence

(*ii*) Raji wants to know the number of functions from *A* to *B*. How many number of functions are possible?

(*a*) 6^2 (*b*) 2^6 (*c*) $6!$ (*d*) 2^{12}

(*iii*) Let *R* be a relation on *B* defined by $R = \{(1,2), (2,2), (1,3), (3,4), (3,1), (4,3), (5,5)\}$. Then *R* is

(*a*) Symmetric (*b*) Reflexive (*c*) Transitive (*d*) None of these three

(*iv*) Raji wants to know the number of relations possible from *A* to *B*. How many numbers of relations are possible?

(*a*) 6^2 (*b*) 2^6 (*c*) $6!$ (*d*) 2^{12}

(*v*) Let $R : B \to B$ be defined by $R=\{(1,1),(1,2), (2,2), (3,3), (4,4), (5,5),(6,6)\}$, then R is

(*a*) Symmetric (*b*) Reflexive and Transitive

(*c*) Transitive and symmetric (*d*) Equivalence

Sol. (*i*) Given $R : B \to B$ be defined by

$R = \{(x, y) : y \text{ is divisible by } x\}$

Reflexive : Let $x \in B$, since x always divide x itself.

$\therefore (x, x) \in R$

It is reflexive.

Symmetric : Let $x, y \in B$ and let $(x, y) \in R$

$\Rightarrow y$ is divisible by x

$\Rightarrow \frac{y}{x} = k_1$, where k_1 is an integer.

$\Rightarrow \frac{x}{y} = \frac{1}{k_1} \neq \text{integer}.$

$\therefore (y, x) \notin R$

It is not symmetric.

Transitive : Let $x, y, z \in B$ and

Let $(x, y) \in R \Rightarrow \frac{y}{x} = k_1$, where k_1 is an integer.

and, $(y, z) \in R \Rightarrow \frac{z}{y} = k_2$, where k_2 is an integer.

$\therefore \quad \frac{y}{x} \times \frac{z}{y} = k_1 . k_2 = k$ (integer)

$\Rightarrow \frac{z}{x} = k \quad \Rightarrow (x, z) \in R$

It is transitive.

Hence, relation is reflexive and transitive but not symmetric.

$\therefore$ Option (*a*) is correct.

(*ii*) We have,

$A = \{S, D\} \Rightarrow n(A) = 2$

and, $B = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(B) = 6$

$\therefore$ Number of functions from A to B is 6^2.

$\therefore$ Option (*a*) is correct.

(*iii*) Given,

R be a relation on B defined by

$R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$

R is not reflexive since $(1, 1), (3, 3), (4, 4) \notin R$

R is not symmetric as $(1, 2) \in R$ but $(2, 1) \notin R$

and, R is not transitive as $(1, 3) \in R$ and $(3, 1) \in R$ but $(1, 1) \notin R$

$\therefore$ R is neither reflexive nor symmetric nor transitive.

$\therefore$ Option (*d*) is correct.

(*iv*) Total number of possible relations from A to $B = 2^{12}$

$\therefore$ Option (*d*) is correct.

(*v*) Given $R : B \to B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$\therefore$ R is reflexive as each elements of B is related to itself and R is also transitive as $(1, 2) \in R$ and $(2, 2) \in R$

$\Rightarrow (1, 2) \in R$

$\therefore$ R is reflexive and transitive.

$\therefore$ Option (*b*) is correct.

3. Read the following and answer any four questions from (*i*) to (*v*).

An organization conducted bike race under 2 different categories-boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ $G=\{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. [*CBSE Question Bank*]

Ravi decides to explore these sets for various types of relations and functions

Answer the questions given below.

(i) Ravi wishes to form all the relations possible from B to G. How many such relations are possible?

(a) 2^6 (b) 2^5 (c) 0 (d) 2^3

(ii) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of same sex$\}$, Then this relation R is

(a) Equivalence

(b) Reflexive only

(c) Reflexive and symmetric but not transitive

(d) Reflexive and transitive but not symmetric

(iii) Ravi wants to know among those relations, how many functions can be formed from B to G?

(a) 2^2 (b) 2^{12} (c) 3^2 (d) 2^3

(iv) Let $R : B \to G$ be defined by $R = \{ (b_1, g_1), (b_2, g_2), (b_3, g_1)\}$, then R is

(a) Injective (b) Surjective

(c) Neither Surjective nor Injective (d) Surjective and Injective

(v) Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

(a) 0 (b) 2! (c) 3! (d) 0!

Sol. We have sets

$B = \{b_1, b_2, b_3\}$, $G = \{g_1, g_2\}$

$\Rightarrow n(B) = 3$ and $n(G) = 2$

(i) Number of all possible relations from B to $G = 2^{3 \times 2} = 2^6$

$\therefore$ Option (a) is correct.

(ii) Given relation R = $\{(x, y) : x$ and y are student of same sex$\}$

On the set B.

Since the set is $B = \{b_1, b_2, b_3\}$ = all boys

$\therefore$ It is ans equivalence relation.

$\therefore$ Option (a) is correct.

(*iii*) We have,

$B = \{b_1, b_2, b_3\} \Rightarrow n(B) = 3$

$G = \{g_1, g_2\} \Rightarrow n(G) = 2$

$\therefore$ Total no. of possible functions from B to $G = 2^3$

$\therefore$ Option (*d*) is correct.

(*iv*) We have,

$R : B \to G$ be defined by

$R = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

It is not injective because $(b_1, g_1) \in R$ and $(b_3, g_1) \in R$

So $b_1 \neq b_3 \Rightarrow$ same image g_1.

It is surjective because its Co-domain = Range.

$\therefore$ R is Surjective.

$\therefore$ Option (*b*) is correct.

(*v*) Since R is not injective therefore number of injective functions = 0

$\therefore$ Option (*a*) is correct.

ASSERTION-REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(*a*) *Both A and R are true and R is the correct explanation of A.*

(*b*) *Both A and R are true but R is not the correct explanation of A.*

(*c*) *A is true but R is false.*

(*d*) *A is false and R is also false.*

1. **Assertion (A) :** Let L be the collection of all lines in a plane and R_1 be the relation on L as $R_1 = \{(L_1, L_2) : L_1 \perp L_2\}$ is a symmetric relation.

   Reason (R) : A relation R is said to be symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$.

2. **Assertion (A) :** Let R be the relation on the set of integers Z given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$ is an equivalence relation.

   Reason (R) : A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

3. **Assertion (A) :** Let $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x$, then f is a one-one function.

   Reason (R) : A function $g : A \to B$ is said to be onto function if for each $b \in B, \exists\, a \in A$ such that $g(a) = b$.

4. **Assertion (A) :** Let function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ be an onto function. Then it must be one-one function.

   Reason (R) : A one-one function $g : A \to B$, where A and B are finite set and having same number of elements, then it must be onto and vice-versa.

Answers

1. (*a*) **2.** (*a*) **3.** (*b*) **4.** (*a*)

HINTS/SOLUTIONS OF SELECTED MCQS

1. Since every element of A is related to itself in the given relation R, therefore R is reflexive and as $(1, 2) \in R$ and $(2, 2) \in R \Rightarrow (1, 2) \in R$ also $(1, 3) \in R$ and $(3, 2) \in R \Rightarrow (1, 2) \in R$. Again $(1, 3) \in R$ and $(3, 3) \in R \Rightarrow (1, 3) \in R$. Thus R is also transitive. Hence relation R is reflexive and transitive but not symmetric because, $(1, 2) \in R$ but $(2, 1) \notin R$, also $(1, 3) \in R$ but $(3, 1) \notin R$ and $(3, 2) \in R$ but $(2, 3) \notin R$.

   Option (b) is correct.

2. On the set $A = \{a, b, c, d\}$ given relation $R = \{(a, b), (b, a), (a, a)\}$ is symmetric and transitive only.

   Since, $(a, b) \in R \Rightarrow (b, a) \in R$, therefore it is symmetric

   Also, $(a, b) \in R$ and $(b, a) \in R \Rightarrow (a, a) \in R$, so it is also transitive. As (b, b), (c, c) and (d, d) does not belong to R hence R is not reflexive.

   Hence relation R is symmetric and transitive only.

   Option (d) is correct.

3. For any $x \in \mathbb{R}$

   $x - x + \sqrt{2} = \sqrt{2}$ is an irrational number $\Rightarrow (x, x) \in R \ \forall \ x \in \mathbb{R}$

   $\therefore$ R is reflexive

   For 2, $\sqrt{2} \in \mathbb{R}$

   $\sqrt{2} - 2 + \sqrt{2} = 2\sqrt{2} - 2$ is an irrational number.

   $\Rightarrow (\sqrt{2}, 2) \in \mathbb{R}$

   But $2 - \sqrt{2} + \sqrt{2} = 2$ which is a rational number

   $\Rightarrow (2, \sqrt{2}) \notin R$

   $\Rightarrow$ R is not reflexive

   R is not transitive

   For 2, $\sqrt{3}$, $\sqrt{2} \in \mathbb{R}$

   $\because$ $2 - \sqrt{3} + \sqrt{2} = 2 - (\sqrt{3} - \sqrt{2})$ is an irrational number

   $\Rightarrow (2, \sqrt{3}) \in R$

   Also $\sqrt{3} - \sqrt{2} + \sqrt{2} = \sqrt{3}$ which is an irrational number

   $\Rightarrow (\sqrt{3}, \sqrt{2}) \in R$

   But $2 - \sqrt{2} + \sqrt{2} = 2$ which is a rational number.

   $\Rightarrow (2, \sqrt{2}) \notin R$

   $\Rightarrow$ R is not transitive

   Option (a) is correct.

4. Given, $aRb \Rightarrow a$ is brother of b

   This does not mean that b is also a brother of a because b can be a sister of a.

   Hence, R is not symmetric.

   Again, $aRb \Rightarrow a$ is brother of b and $bRc \Rightarrow b$ is brother of c.

   So, a is brother of c.

   Hence, R is transitive.

   Option (b) is correct.

5. We are given set A = {1, 2, 3}

Number of equivalance relation on A = number of possible portion of {1, 2, 3}

i.e., $3 = 1 + 1 + 1$ Only one combination

$3 = 1 + 2$ 3 Possible combination

$3 = 3$ 1 possible combination

i.e., (*i*) {{1}, {2}, {3}} *i.e.*, {(1, 1), (2, 2), (3, 3)}

(*ii*) {{1, 2}, {3} *i.e.*, {(1, 2), (2, 1), (3, 3)}

(*iii*) {{1, 3}, {2}} *i.e.*, {(1, 3), (3, 1), (2, 2)}

(*iv*) {{2, 3}, {1}} *i.e.*, {(2, 3), (3, 2), (1, 1)}

(*v*) {{1, 2, 3}} *i.e.*, {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

i.e., Total number of equivalence relation = 5

Option (*d*) is correct.

6. For $l, m \in L$

if $(l, m) \in R \Rightarrow l \perp m \Rightarrow m \perp l \Rightarrow (m, l) \in R$

$\therefore$ R is symmetric.

Option (*b*) is correct.

7. Required relation is reflexive and symmetric but not transitive is given by

$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3)\ (3, 1)\}$

which is reflexive as $(a, a) \in R\ \forall\ a \in A$

which is symmetric as $(a, b) \in R \Rightarrow (b, a) \in R$ for $a, b \in A$

But $(2, 1), (1, 3) \in R \not\Rightarrow (2, 3) \in R$

Hence R is not transitive.

There is only one such relation.

Option (*a*) is correct.

9. We have $n(A) = m$, $n(B) = n$.

$\therefore$ Number of relations defined from A to B

= number of possible subsets of A × B $= 2^{n(A \times B)} = 2^{mn}$

Option (*a*) is correct.

10. The total number of injective mappings from the set containing n elements into the set containing m elements is ${}^mP_n$. So here it is ${}^4P_3 = 4! = 24$.

Option (*c*) is correct.

11. We have $f : \mathbb{R} \to \mathbb{R}$: such that

$$f(x) = 2^x + 2^{|x|} = \begin{cases} 2^x + 2^x & \text{if } x \geq 0 \\ 2^x + 2^{-x} & \text{if } x < 0 \end{cases} = \begin{cases} 2^{x+1} & \text{if } x \geq 0 \\ 2^x + 2^{-x} & \text{if } x < 0 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} 2^{x+1} \log 2 & \text{if } x \geq 0 \\ 2^x \log 2 + (-2^{-x} \log 2) & \text{if } x < 0 \end{cases} = \begin{cases} 2^{x+1} \log 2 & \text{if } x \geq 0 \\ \log 2\,(2^x - 2^{-x}) & \text{if } x < 0 \end{cases}$$

$\because$ $f'(x) > 0\ \forall\ x \geq 0$ and $f'(x) < 0\ \forall\ x < 0$

$\Rightarrow f(x)$ is strictly increasing in $\mathbb{R} \Rightarrow f(x)$ is one-one.

Also, $f(x) \to \infty$ if $x \to \pm\infty$ and $f(x) > 0\ \forall\ x \in \mathbb{R}$

$\Rightarrow f$ is an into function.

$\therefore$ $f(x)$ is one-one and into function.

Option (c) is correct.

12. We have $n(A) = 5$ and $n(B) = 6$

$\therefore$ Number of one-one mopping from A to $B = 6!$

As $n(A) < n(B)$

$\Rightarrow$ There is no onto function from A to B. *i.e.*, number of onto function = 0.

$\therefore$ Number of one-one and onto functions from A to $B = 0$

Option (c) is correct.

13. $f: \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x + 2$

One-one For $x_1, x_2 \in \mathbb{Z}$ such that $x_1 \neq x_2 \quad \Rightarrow \quad x_1 + 2 \neq x_2 + 2$

$\Rightarrow f(x_1) \neq f(x_2)$

$\Rightarrow f$ is one-one.

Onto Let $y \in \mathbb{Z}$ (co-domain) such that

$f(x) = y \quad \Rightarrow \quad x + 2 = y \quad \Rightarrow \quad x = y - 2$

For $y \in \mathbb{Z}$ (co-domain), $\exists\ x = y - 2 \in \mathbb{Z}$ (domain) such that

$f(x) = f(y - 2) = y - 2 + 2 = y$

$\Rightarrow f$ is onto

As f is one-one and onto.

$\Rightarrow f$ is a bijective function.

Option (b) is correct.

14. Given that, $f(x) = x^2 - 4x + 5$

Let $y = x^2 - 4x + 5$

$y = x^2 - 4x + 4 + 1 = (x - 2)^2 + 1$

$(x - 2)^2 = y - 1 \quad \Rightarrow \quad x - 2 = \sqrt{y - 1}$

$\Rightarrow \quad x = 2 + \sqrt{y - 1}$

$\therefore \quad y - 1 \geq 0, y \geq 1$

Range = $[1, \infty)$

Option (b) is correct.

15. Let $f(x) = 17 \Rightarrow x^2 + 1 = 17$

$\Rightarrow \quad x = \pm 4 \Rightarrow$ Pre image of 17 are $\{4, -4\}$

and let $f(x) = -3 \Rightarrow x^2 + 1 = -3 \quad \Rightarrow \quad x^2 = -4$ which is not true and hence -3 has no pre image

17. $\because b > 6$ and $a = b - 2$

$\Rightarrow (6, 8) \in R$ as $8 > 6$ and $6 = 8 - 2$

Option (c) is correct.

18. f is not one-one because

$f(-2) = (-2)^4 = 16$

$f(2) = (2)^4 = 16$

i.e., -2 and $2 \in R$ (Domain) have same f-image in R (co-domain)

$\Rightarrow f$ is not one-one.

Also $f(x) = x^4$ never achieve negative value.

$\Rightarrow$ All negative real number of co-domain R have no pre-image in Domain R.

$\Rightarrow$ f is not onto.

Hence, f is neither one-one nor onto.

Option (d) is correct.

19. f is one-one because

$$f(x_1) = f(x_2) \Rightarrow 3x_1 = 3x_2$$

$\Rightarrow \quad x_1 = x_2 \qquad \forall\, x_1, x_2 \in R \quad$ (Domain)

Also f is onto as

Let $f(x) = y \quad \Rightarrow \quad 3x = y \quad \Rightarrow \quad x = \dfrac{y}{3}$

$\forall\, y \in R$ (codomain) $\exists x = \dfrac{y}{3} \in R$ (domain)

such that $f(x) = f\left(\dfrac{y}{3}\right) = 3 \times \dfrac{y}{3} = y$

$\Rightarrow \quad f(x)$ is onto

Therefore, f is one-one onto.

Option (a) is correct.

21. Let x be the pre image of 5

$\Rightarrow f(x) = 5$

$\Rightarrow x^2 + 1 = 5$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

i.e., pre-image of 5 is $-2, +2$.

Similarly if x be pre-image of -5

$\Rightarrow f(x) = -5$

$\Rightarrow x^2 + 1 = -5$

$\Rightarrow x^2 = -6$

$x = \pm\sqrt{-6} \notin R$

i.e., No real number is pre-image of -5. Hence ϕ is the primage of -5.

Option (c) is correct.

22. To find out the domain of f, we have to find out that value of x for which $f(x)$ is real.

$\Rightarrow \quad x^2 - 4 \geq 0$

$\Rightarrow \quad (x+2)(x-2) \geq 0$

$\Rightarrow \quad (x+2) \geq 0, (x-2) \geq 0$ or $(x+2) \leq 0, (x-2) \leq 0$

$\Rightarrow \quad x \geq -2, x \geq 2$ or $x \leq -2, x \leq 2$

$\Rightarrow \quad x \geq 2$ or $x \leq -2$

Domain of f is $(-\infty, -2] \cup [2, \infty)$

Option (c) is correct.

23. $f(-2) + f(0) + f(2) + f(5) = -2 + 0 + 4 + 15 = 17$

Option (b) is correct.

24. As R is reflexive relation on A, and for being reflexive $(a, a) \in R, \forall\, a \in A$

Therefore, the minimum number of ordered pair in R is n.

$\Rightarrow m \geq n$.

Option (a) is correct.

25. Given function is $f(x) = \log_{3+x}(x^2 - 1)$

It is obvious that $f(x)$ is defined when $x^2 - 1 > 0$, $3 + x > 0$ and $3 + x \neq 1$.

Now, $x^2 - 1 > 0 \Rightarrow x^2 > 1$

$\Rightarrow x < -1$ or $x > 1$

$3 + x > 0 \Rightarrow x > -3$

$3 + x \neq 1 \Rightarrow x \neq -2$

Therefore, domain of the function $f(x) = (-3, -2) \cup (-2, -1) \cup (1, \infty)$

Option (c) is correct.

26. From definition of onto function,

Range of function = Codomain of function = $[0, \frac{\pi}{2})$

$\Rightarrow \quad 0 \leq \tan^{-1}(x^2 + x + a) < \frac{\pi}{2}$

$\rightarrow \quad 0 \leq (x^2 + x + a) < \infty$

$\Rightarrow \quad x^2 + x + a > 0 \ \forall x \in R$

Hence $D \leq 0$

$\Rightarrow \quad 1^2 - 4a \leq 0$

$\Rightarrow \quad 4a \geq 1$

$\Rightarrow \quad a \geq \frac{1}{4}$

$\Rightarrow \quad a \in [\frac{1}{4}, \infty)$

Option (b) is correct.

27. We have,

$f : R \to R$ and $g : R \to R$ are such that

$$f(x) = \begin{cases} 0, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

and $$g(x) = \begin{cases} 0, & x \in \text{irrational} \\ x, & x \in \text{rational} \end{cases}$$

$\therefore (f - g) : R \to R$ such that,

$$(f - g)(x) = \begin{cases} -x, & x \in \text{rational} \\ x, & x \in \text{irrational} \end{cases}$$

From definition of $(f - g) : R \to R$, it is obvious that, each rational number of domain of $(f - g)(x)$, associate to its negative rational number in codomain/range and each irrational number of domain of $(f - g)(x)$, associate to same irrational number in codomain/range.

$\Rightarrow$ For each $x \in$ Domain of $(f - g)(x)$, there is only one value in codomain/range of $(f - g)(x)$.

Hence, $(f - g)(x)$ is one-one onto.

Option (a) is correct.

28. We have set $A = \{1, 2, 3, 4\}$ & relation

$R = \{(1, 2), (3, 4)\}$ on A

As for $(a, b) \in R, \nexists (b, c) \in R$ such that

$(a, c) \notin R$.

Hence R is transitive.

So (*b*) is correct option.

Option (*b*) is correct.

29. $\because |A| = 4, |B| = 5$ so there does not exist.

one-one and onto $\because |B| > |A|$ so it is not onto.

So (*a*) is correct option.

Option (*a*) is correct.

30. Number of onto functions are given by

$2^3 - {}^2C_1 (2-1)^3 + {}^2C_2 (2-2)^3$

$= 8 - 2 \times 1 + 0 = 8 - 2 = 6$

Option (*c*) is correct.

31. Number of one-one function $= {}^6P_4$

$= \frac{\underline{|6}}{\underline{|2}} = \frac{720}{2} = 360$

Option (*c*) is correct.

32. Number of one-one and onto function from A to B where $|A| = m$ is $\underline{|m}$.

$\therefore$ Number of one-one onto function $= \underline{|4} = 24$

Option (*b*) is correct.

[**Note:** One-one onto function (bijective) from A to B is possible if A and B have same number of elements.]

34. Let R be a relation on the set of all intergers $\mathbb{Z}$, defined by

$aRb \Leftrightarrow a > b \; \forall \; a, b \in \mathbb{Z}$

(*i*) **Reflexive:** For $1 \in \mathbb{Z}$

$1 \not R 1$ as $1 \not> 1$ so $(1, 1) \notin R \Rightarrow R$ is not reflexive on $\mathbb{Z}$

(*ii*) **Symmetric:** $(3, 2) \subset R$ as $3 > 2$

But $(2, 3) \notin R$ as $2 \not> 3$

Hence R is not symmetric on $\mathbb{Z}$

(*iii*) **Transitive:** Let $(a, b) \in R$ and $(b, c) \in R$ $a > b$ and $b > c$

Now $a > b > c \Rightarrow a > c \Rightarrow (a, c) \in R$

Hence R is a transitive relation on $\mathbb{Z}$

Option (*c*) is correct.

36. $f(x) = 4^x + 4^{|x|}$

One-one

Let $x_1, x_2 \in R$ (domain) such that

$x_1 \neq x_2$

$\Rightarrow 4^{x_1} + 4^{|x_1|} \neq 4^{x_2} + 4^{|x_2|}$

$\Rightarrow f(x_1) \neq f(x_2)$

f is one-one

Onto

For $0 \in R$ (Co-domain) there is no $x \in R$ (domain) such that $f(x) = 0$.

$\therefore$ f is not onto

Range of $f = R - \{0\} \subseteq R$

Hence f is one-one into function.

Option (a) is correct.

39. Here co-domain $= \left[0, \frac{\pi}{2}\right)$

For onto function, we have

Co-domain = Range $= 0 \leq x < \frac{\pi}{2}$

This is valid if $x^2 + x + 2a \geq 0$

[$\because f(x) \geq 0$ *i.e.* $Ax^2 + Bx + C \geq 0$ then $D \leq 0$ if $A > 0$.]

i.e., $x^2 + x + 2a \geq 0 \Rightarrow 1^2 - 4 \times 1 \times 2a \leq 0$

$\rightarrow 1 - 8a \leq 0 \rightarrow 1 \leq 8a \rightarrow 8a \geq 1 \rightarrow a \geq \frac{1}{8}$

$\therefore \; a \in \left[\frac{1}{8}, \infty\right)$

Option (d) is correct.

40. We are given that

$$(f(x))^2 - 4f(x)f'(x) + (f'(x))^2 = 0$$

$$\Rightarrow f'(x) = \frac{4f(x) \pm \sqrt{16(f(x))^2 - 4(f(x))^2}}{2}$$

$$\therefore f'(x) = \frac{4f(x) \pm 2f(x)\sqrt{4-1}}{2}$$

$$= \frac{4f(x) \pm 2\sqrt{3}f(x)}{2}$$

$$= f(x)(2 \pm \sqrt{3})$$

$$\Rightarrow \frac{f'(x)}{f(x)} = (2 \pm \sqrt{3})$$

Integrating, we get

$\Rightarrow \log f(x) = (2 \pm \sqrt{3})x + C$

$\Rightarrow f(x) = e^{(2 \pm \sqrt{3})x + C} = e^C e^{(2 \pm \sqrt{3})x} = \lambda e^{(2 \pm \sqrt{3})x}$

where $\lambda = e^C$

$\Rightarrow f(x) = \lambda e^{(2 \pm \sqrt{3})x} = \lambda e^{(2 + \sqrt{3})x}, \lambda e^{(2 - \sqrt{3})x}$

Option (c) is correct.

41. $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = 2\tan^{-1}x$

$f(x)$ is one-one and onto

i.e., $f'(x) > 0$ or $f'(x) < 0$ and co-domain = range of $f(x)$

$B = f(-1, 1) = (2\tan^{-1}(-1), 2\tan^{-1}(1))$

$$= \left(2 \times \left(-\frac{\pi}{4}\right), 2 \times \frac{\pi}{4}\right) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Option (*c*) is correct.

42. Let $f(x) = y = \dfrac{x}{1+|x|} \; \forall x \in \mathbb{R}, y \in \mathbb{R}$

$\therefore f(x) = \dfrac{x}{1+x}$ or $\dfrac{x}{1-x}$ is one-one

Here range of $f(x)$ is $R - \{-1, 1\}$

But y can not have any of the values $-1, 1$ for some x.

$\therefore f(x)$ is not an onto function.

Option (*c*) is correct.

44. Number of onto function

$$= 3^6 - {}^3C_1 (3-1)^6 + {}^3C_2 (3-2)^6 - {}^3C_3 (3-3)^6$$

$$= 3^6 - 3 \times 2^6 + 3 \times 1 = 3^6 - 3 \times 2^6 + 3$$

$$= 3 \times (3^5 - 2^6 + 1) = 3(243 - 64 + 1)$$

$$= 3 \times (244 - 64) = 3 \times 180 = 540$$

Option (*d*) is correct.

45. As $(7, 7) \notin R$, so R can not be reflexive

Again $(7, 8) \in R$ but $(8, 7) \notin R$, so R is not symmetric.

As $(7, 8), (8, 8) \in R \Rightarrow (7, 8) \in R \Rightarrow R$ is transitive.

Option (*a*) is correct.

46. Let $A = \{1, 2, 3, ..., 12\}$, $n(A) = 12$ (say m)

$B = \{1, 2, 3, ..., 11\}$, $n(B) = 11$ (say n)

$\therefore$ Total number of function from A to $B = 11^{12}$

$\therefore$ Number of onto functions from A to B = $\sum\limits_{r=1}^{n} (-1)^{n-r} \; {}^nC_r r^m$

= coefficient of x^m in $m!\,(e^x - 1)^n$ ...(*i*)

Putting $m = 12$, $n = 11$ and $r = 1, 2, 3, ..., 11$. The number of onto functions is given by

$= (-1)^{11-1}\,{}^{11}C_1\, 1^{12} + (-1)^{11-2}\,{}^{11}C_2 2^{12} + (-1)^{11-3}\,{}^{11}C_3 3^{12}$

$+ ... + (-1)^1\,{}^{11}C_{10} 10^{12} + (-1)^0\,{}^{11}C_{11}\, 11^{12}$

$= {}^{11}C_{11}\, 11^{12} - {}^{11}C_{10} 10^{12} + {}^{11}C_9 9^{12} + ... + {}^{11}C_3\, 3^{12} - {}^{11}C_2\, 2^{12} + {}^{11}C_1\, 1^{12}$

$= {}^{11}C_0\, 11^{12} - {}^{11}C_1 10^{12} + {}^{11}C_2 9^{12} + ... + {}^{11}C_8 3^{12} - {}^{11}C_9 2^{12} + {}^{11}C_{10} 1^{12}$

Also, coefficient x^{12} in $12!\,(e^x - 1)^{11}$

= coefficient of x^{12} in $12!\left(1 + \dfrac{x}{1!} + \dfrac{x^2}{2!} + ... + \infty - 1\right)^{11}$

= coefficient of x^{12} in $12!\left(\dfrac{x}{1!} + \dfrac{x^2}{2!} + ... + \infty\right)^{11}$

= coefficient of x^{12} in $12!\, x^{11}\left(\dfrac{1}{1!} + \dfrac{x}{2!} + \dfrac{x^2}{3!} + ... + \infty\right)^{11}$

= coefficient of x in $12! \times \left[1 + \left(\dfrac{x}{2!} + \dfrac{x^2}{3!} + ... + \infty\right)\right]^{11}$

= **coefficient of** x **in** $12!\left[{}^{11}C_0 1 + {}^{11}C_1\left(\frac{x}{2!}+\frac{x^2}{3!}+\ldots.+\infty\right)+{}^{11}C_2\left(\frac{x}{2!}+\frac{x^2}{3!}+\ldots.+\infty\right)+\ldots\right]$

= **coefficient of** x **in** $12!\left[{}^{11}C_1\left(\frac{1}{2!}\right)\right]=\frac{12!\times 11}{2!}$

Total number of functions which are not onto $= 11^{12}-\frac{12!\times 11}{2}$

Option (*d*) is correct.

47. $\because f(x\ f(y)) = x^a y^b \qquad \ldots(i)$

Replacing x by $\frac{1}{f(y)}$, we have from (*i*)

$$f(x\times f(y)) = f\left(x\times\frac{1}{x}\right)=\left(\frac{1}{f(y)}\right)^a y^b$$

$$f(1)=\frac{y^b}{(f(y))^a}\Rightarrow (f(y))^a=\frac{y^b}{f(1)}$$

$$\therefore \text{ Put } y = 1, (f(1))^a=\frac{1^b}{f(1)}=\frac{1}{f(1)}$$

$$\Rightarrow (f(1))^{a+1} - 1$$

$$\Rightarrow f(1) = 1^{\left(\frac{1}{a+1}\right)} = 1$$

$$\Rightarrow f(1)=\frac{y^b}{(f(y))^a}=1\Rightarrow (f(y))^a=y^b$$

$$\Rightarrow f(y)=y^{b/a}$$

Replacing y as x, we have

$f(x) = x^{b/a} \qquad \ldots(ii)$

$\therefore f(x\ .\ y^{b/a}) = x^a y^b$

Let $y^{b/a} = t \Rightarrow y = t^{a/b}$

$f(x\ .\ t) = x^a t^a \Rightarrow f(x) = x^a \qquad \ldots(iii)$

Now from (*ii*) and (*iii*), we get

$$x^{b/a}=x^a\Rightarrow\frac{a}{b}=\frac{1}{a}\Rightarrow b=a^2$$

Option (*c*) is correct.

51. Number of relations that can be defined on $A = 2^{n^2}$

Option (*c*) is correct.

54. We have $R = \{(a, a), (b, b), (c, c), (b, c), (a, b)\}$

For $(b, c) \in R$, but $(c, b) \notin R$.

Hence R is not symmetric.

Also for $(a, b), (b, c) \in R$ but $(a, c) \notin R$.

$\Rightarrow$ R is not transitive.

As $(a, a) \in R\ \forall\ a \in A$

Hence R is reflexive.

Option (*c*) is correct.

55. Let $A = \{1, 2\}$, $B = \{a, b\}$

Let $R = \{(1, a), (1, b), (2, a), (2, b)\}$

Clearly R is a relation from A to B

But R is not a function.

As $(1, a), (1, b) \in R$ and $(2, a), (2, b) \in R$

Option (b) is correct.

56. $R = \{(1, 2)\}, A = \{1, 2, 3\}$

Clearly R is neither reflexive nor symmetric.

As $(1, 2) \in R$ but $\nexists$ $(2, b) \in R$ for $b \in A$ such that $(1, b) \notin R$.

Hence R is a transitive relation on A.

Option (b) is correct.

57. $R = \{(a, b) : a \geq b\}$

Reflexive

Clearly $(a, a) \in R \ \forall a \in R$.

Hence R is reflexive.

Symmetric

$\because (2, 1) \in R$ but $(1, 2) \notin R$

Hence R is not symmetric.

Transitive

Let (a, b) and $(b, c) \in R$

$\Rightarrow \quad a \geq b$ and $b \geq c$

$\Rightarrow \quad a \geq c$

Hence (a, b) and $(b, c) \in R \Rightarrow (a, c) \in R$

$\Rightarrow \quad R$ is a transitive relation on R.

Option (b) is correct.

59. $R = \{(x, y) : |x^2 - y^2| < 7\}$

$R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 2), (2,3)\}$

Option (d) is correct.